Transient Magnetotellurics with Adaptive Polarization Stacking

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Summary

Thunderstorm activity produces large amounts of electromagnetic energy which is trapped within the earth-ionosphere waveguide. The random sum of energy from activity on a near global scale produces a low level, quasi-continuous source field. Very large, or equivalently, relatively nearby lightning discharges produce *individual* transient events whose amplitude are significantly larger than that of the low level background field. Therefore, substantial increases in signal-to-noise ratio can be realized by recording exclusively sources of a transient nature. However, the transient events are strongly linearly polarized, the polarization diversity of which can affect the estimation of earth response curves.

We introduce a method by which an adaptive time domain averaging of the transient waveforms is conducted. In its static form, this was termed polarization stacking (Kosteniuk and Paulson, 1988).

Our theoretical analysis has shown that the bias in the impedance tensor or magnetic field tipper estimate converges to zero super-exponentially in stacked signal-to-noise ratio.

Verification of our theoretical analysis was accomplished with Monte-Carlo simulation using real data. It is found that, given typical polarization characteristics of transient sources, adaptive polarization stacking outperforms conventional least-squares and remote-reference analyses in the rate of bias convergence.

Introduction

Magnetotellurics (MT) makes use of naturally occurring fluctuations in the earth's geomagnetic field, along with electric field fluctuations induced within the earth by the former, to map sub-surface resistivity. The chief source of naturally occurring energy in the ELF/VLF [1] bandwidth is due to lightning discharges (Pierce, 1977. Volland, 1982). Thunderstorm activity on a near global scale gives rise to a low level, quasi-continuous component, superimposed on which are *individual* transients which arise from either relatively nearby and/or very large current-moment lightning discharges (Tzanis and Beamish, 1987. Jones and Kemp, 1971). Note that nearby is defined relative to global waveguide attenuation. For example, nearby at 100 Hz may be 6 Mm [2] whereas at 5 kHz, perhaps 1.5 Mm.

Both energy sources can be used to estimate the impedance tensor or magnetic field tipper, but substantial increases in signal-to-noise ratio (SNR) are afforded by attempting to exclusively record transients. The reason is two-fold; firstly, for most of the ELF/VLF bandwidth, there can be significant lengths of time over which the level of continuing activity is near or below the instrumentation noise floor. Therefore, by only recording data when transients are present, the temporal periods of low SNR are avoided. Secondly, transients are generally more than an order of magnitude larger than the low level continuing component.

However, transient data exhibits strong linear polarization. The angle between sources affects the rate of bias convergence with 90° being ideal for the adaptive polarization stacking (APS) method. In addition to the usual sources of bias for the conventional least squares (LS) and remote-reference (RR) techniques, the presence of large amplitude, linearly polarized transients further affects the rate of bias convergence. For LS, the angle between sources can produce bias additional to that caused by finite SNR data. For RR (Gamble et al., 1979), the requirement of circular polarization in the local and reference source fields is violated. As with LS, the angle between sources also affects the rate of bias convergence with RR.

Therefore, we implement an adaptive time domain averaging of the transient waveforms to enhance SNR and thereby reduce bias. Prior to averaging though, we need to group the transients into two distinct sets in accordance with the stack directions defined. Placing the stacks rigidly along the (x, y) co-ordinate system axes results in what was termed polarization stacking (Kosteniuk and Paulson, 1988). As an improvement, we let the orientation of the stacks be adaptive to the constantly changing bearing and amplitude characteristics of the transient data.

In order to realize the ideal $\sqrt{N}$ enhancement in SNR with signal averaging, one requires that the noise be uncorrelated across records and that the same underlying signal be present in every noisy recording. With lightning transients, each received waveform reflects the amount of attenuation and dispersion suffered in propagating to the measurement location. Therefore, each transient is indeed different, but they all have a well defined extremum, each of which can be aligned and subsequently summed in phase, resulting in an SNR enhancement. The improved SNR (ISNR) is bounded below by SNR and above by $\sqrt{N}*$SNR.

Theory

The fundamental quantity of interest for MT surveys is

[1] ELF:Extremely-Low Frequency, 3 Hz - 3 kHz; VLF: Very-Low Frequency, 3 kHz - 30 kHz

[2] 1 Mm =1000 km

the impedance tensor $\tilde{\mathbf{Z}}$ which is the transfer function between mutually orthogonal, horizontal components of magnetic and electric fields as defined in equation (1). A right handed co-ordinate system is typically defined as ^+x North, ^+y East and ^+z vertically down.

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{xx} & \tilde{Z}_{xy} \\ \tilde{Z}_{yx} & \tilde{Z}_{yy} \end{bmatrix} \cdot \begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \tag{1}$$

or simply

$$\tilde{\mathbf{E}} = \tilde{\mathbf{Z}}\tilde{\mathbf{H}}$$

The regression problem is then one of estimating the complex values of $\tilde{\mathbf{Z}}$. This has traditionally been done by minimizing the sum square error of the residual on the electric field channels. The least squares formula (LS) in this case are obtained by post-multiplying both sides of equation (1) by $(\tilde{H}_x^*, \tilde{H}_y^*)$ where the asterisk denotes complex conjugation. Summing over N records we obtain the familiar least squares form in the electric field noise minimization case.

We note that since we have four complex unknowns and only two equations, we need at least two independent measurements of equation (1) to solve for $\tilde{\mathbf{Z}}$. If we consider this simplest case, we have

$$\tilde{E}_{xi} = \tilde{Z}_{xx}\tilde{H}_{xi} + \tilde{Z}_{xy}\tilde{H}_{yi} \tag{2}$$

$$\tilde{E}_{yi} = \tilde{Z}_{yx}\tilde{H}_{xi} + \tilde{Z}_{yy}\tilde{H}_{yi}$$

$$i = 1, 2.$$

Solving this pair of 2 × 2 linear systems yields the two-point formulas for $\tilde{\mathbf{Z}}$. The modifier two-point is used as although these are complex variables, the solution is analogous to passing a plane through the origin and two other points.

Until now, statistical analysis of the bias in estimates of $\tilde{\mathbf{Z}}$ has been done for the LS (Sims et al., 1971) and remote-reference (RR) (Gamble et al, 1979) solutions, but never for the two-point solution. Although it was claimed that RR is "unbiased"(Gamble et al., 1979), this is valid only in the limit of infinitely many independent measurements. Practically then, the solutions of $\tilde{\mathbf{Z}}$ just mentioned in fact display a finite *convergence* of bias as SNR becomes large. However, for RR and APS, there is also a bias convergence to some arbitrarily small level at fixed SNR, as the number of measurements N becomes large.

The LS solution has a bias that converges very slowly as SNR^{-2}. By contrast, the bias in the two-point formula is due only to nonlinearity in the complex quotient. Its bias is of infinitely smaller order, namely $exp(-\frac{1}{2}(ISNR\sin(\alpha))^2)$, where α is the angle between the stacked events, with $0 \leq \alpha \leq 90°$.

However, the polarization diversity of received transients may be such that α is much less than the optimal 90°, but even for $\alpha = 30°$, the reduction factor of $\sin(\alpha) = 0.5$ is quickly offset by the improvement in SNR. At only moderate ISNR and angle α, the stacking bias can already be less than $10^{-7}|\tilde{\mathbf{Z}}|$ and hence negligible in single precision.

We use complex function theory to give an easy proof of the exactness of our bias formula in the one-dimensional case. We then extend the computation to the full tensor of equation (1). As the formula becomes more complicated, we simplify the bias expression to that of an upper bound of the form, shown for $\tilde{Z}_{xy}$ only,

$$| < \tilde{Z}_{xy} > - \tilde{Z}_{xy}^{true}| \leq A(ISNR, \alpha) + U(ISNR, \alpha), \tag{3}$$

which is well-approximated as simply

$$| < \tilde{Z}_{xy} > - \tilde{Z}_{xy}^{true}| \leq U(ISNR, \alpha), \tag{4}$$

where $< \tilde{Z}_{xy} >$ is the ensemble average over all noise instances. Here $U(ISNR, \alpha)$ is a numerically evaluated upper bound for a certain probability integral, while $A(ISNR, \alpha)$ is analytically known and small by comparison since it contains a factor of $exp(-\frac{1}{2}(ISNR\sin(\alpha))^2)$. Yet the bound $U(ISNR, \alpha)$ still has "super-exponential" convergence, as its profile on a log plot shows.

Monte-Carlo Simulation with Real Data

To confirm the super-exponential bias convergence of the APS method, and to analyze both LS and RR techniques, Monte-Carlo simulations were performed with transient data.

To make the simulations as realistic as possible, magnetic field data recorded on Sept 14, 2000 were used with a noise-free $\tilde{\mathbf{Z}}$ obtained from modeling a one-dimensional earth. The electric field data which perfectly corresponds to the magnetic field data was created through equation (1) using the synthetic one-dimensional $\tilde{\mathbf{Z}}$. The magnetic field data are typical for a fall recording in south-central Saskatchewan (Goldak, 1998) with two dominant sources, one in the Great Lakes region approximately 2.2 Mm distant, the other in the Gulf of Mexico approximately 3 Mm distant (www.lightningstorm.com). The perfect mapping between magnetic and electric field data sets was then disrupted by introducing linearly additive, normally distributed, pseudo-random noise in varying amounts so as to vary the SNR. For a given SNR, this process was repeated between 1×10^5 to 12×10^6 times to generate a noisy family of impedance tensor curves from which an estimate of the bias and its error were found.

We also carried out Monte-Carlo simulations with synthetic transient data generated by using the Bruce and Golde (1941) current model, the modal waveguide equations of Wait (1962) and the waveguide transmission function data of Barr (1970). However, we felt that by using real data the largest amount of variablilty in the time domain waveforms would be realized, thus testing the bias convergence of the APS method in the most realistic manner.

Shown in Figure 1 is the theoretical upper bound on the bias for a 30° stack angle, and the bias convergence as estimated by Monte-Carlo simulation for a 20° stack angle. Note that the theoretical analysis is in terms of ISNR, therefore, pre-stack Monte-Carlo SNR was converted to approximate ISNR by multiplying the former by $\sqrt{N}$, where N is the number of events in each stack. Of course the true SNR enhancement will be something less than $\sqrt{N}$, no matter what it truly is, we see that the theoretical upper bound is still very generous in this case and that the order of convergence is verified by the results of the Monte-Carlo simulation.

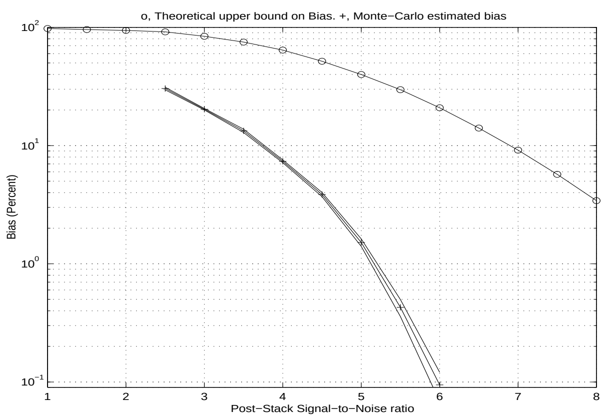

Figure 1: Confirmation of super-exponential Bias Convergence

Shown in Figure 2 is a comparison of the rate of bias convergence between APS, LS and RR. Eight transient events were used with a stack angle of 20°. We see that the bias in the APS method converges faster than either LS or RR techniques. Interestingly, LS bias convergence is near exponential. Theory predicts that for large event count and SNR that the LS bias convergence should be only algebraic. This remains to be confirmed through further Monte-Carlo simulation.

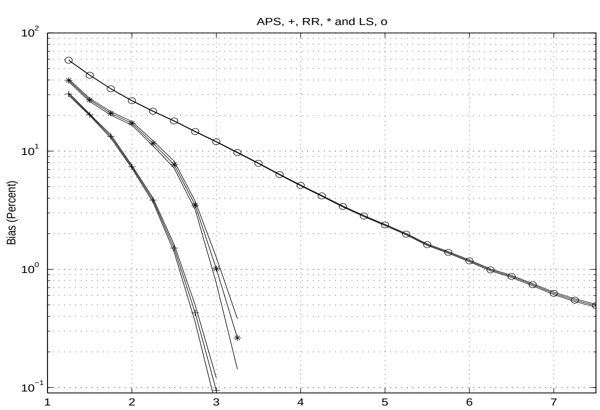

Figure 2: Bias Convergence

Shown in Figure 3 is the bias convergence at constant SNR and approximately constant stack angle of 20°, but with varying number of events. This is a more realistic situation as we have control over the number of recorded events during field work. In agreement with RR theory, the bias in the RR estimate is a partly a function of the sample size. Interestingly, the bias in the LS estimate worsens slightly as the number of events increases. Once again, the bias convergence of the APS method is faster than both RR and LS.

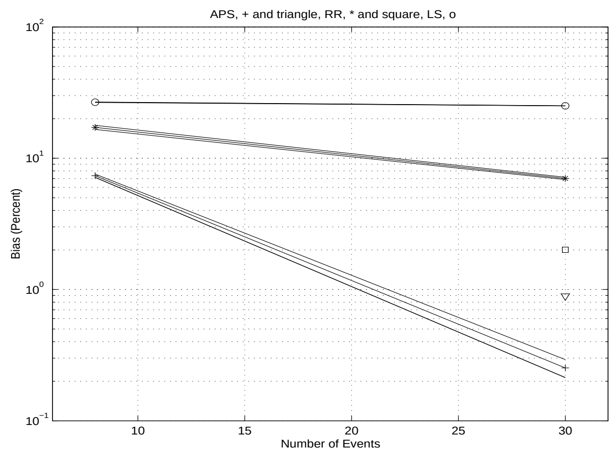

Figure 3: Bias Convergence with increasing N

Also shown in Figure 3 is the bias for the "scattered bearing" case. APS bias is indicated by the triangle and RR by the box. This was done in an attempt to compare most fairly RR and APS. For the latter, each event is projected and averaged along the mean bearing of its stack. Therefore, having two well collimated sources results in a very good SNR enhancement as every event lies very close to the mean bearing of its stack. If we instead had a diffuse scatter of bearings, given equal amplitude, events which are farther away from the mean direction of their stack contribute less to the overall SNR enhancement than do ones which lie closer. The factor being $\approx \cos(\theta)$ where θ is the difference between the stack and event bearing. Conversely, the "scattered bearing" case is advantageous for RR, which enjoys any bearing distribution that more closely approximates circular polarization. However, we see that even in this case, APS outperforms RR, although the improvement ratio is much smaller.

Conclusions

The largest naturally occurring signals in the ELF/VLF bandwidth are transients. In order to record transients most efficiently, a time localized recording technique is desired. In so doing, a substantial SNR enhancement is realized as the temporal periods of low SNR are completely avoided.

We have shown both theoretically and practically, that an adaptive time domain averaging of transient waveforms results in essentially unbiased estimates of the impedance tensor or magnetic field tipper while using only four channels of data. We do require that the noise between separate records on any given channel be uncorrelated. Our analysis has further shown that when one works exclusively with transients, given typical polarization characteristics, the bias convergence of the APS method is of higher order than LS or RR.

Towards more closely achieving the ideal $\sqrt{N}$ enhancement in SNR, we are working on a frequency domain version of the APS method. It is hoped that this will make even better use of the transient data by rectifying the issue of non similarity of time domain waveforms.

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